

# Particle Flow in Turbomachinery with Application to Laser-Doppler Velocimetry

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## Theme

**L**ASER-Doppler velocimetry (LDV) is a noncontact, optical technique for determining the velocity of flowing fluids by measuring the velocity of tracer particles entrained in the fluid rather than by measuring the velocity of the fluid itself. The accuracy of the LDV system is limited by the accuracy with which the naturally occurring or artificially generated tracer particles follow the fluid flow. In order to help identify application guidelines of the LDV technique for the aircraft turbomachinery field, a theoretical analysis was conducted of the dynamic behavior of micron-size particles entrained in high Mach number subsonic gas flow passing through a stationary cascade of turbine stator blades. The particle-to-gas velocity ratio and particle angular deviation relative to the gas were determined as functions of particle size and mass density. Results for the blade configuration examined indicated that velocity and angular deviations generally less than 1% and 0.5°, respectively, could be achieved with 1 g/cc tracer particles with diameters of 0.5  $\mu$  or less.

## Contents

The flow system analyzed in this study is the gas-particle flow on the mean blade-to-blade surface of a circular stationary, nearly axial flow, cascade of turbine stator blades. The cascade has 72 blades and each has a 3.85 in. span and 2.47 in. chord. The turbine tip diameter is 31.9 in., the hub-to-tip radius ratio is 0.76, and the mean radius solidity is 2.05. The flow is considered to be two-dimensional and the independent variables are the meridional streamline distance  $X$  and the tangential angle  $\theta$ . A two-dimensional analysis is justified since the meridional streamline radius is essentially constant. For purposes of a finite-difference analysis of the governing equations, a solution region with a meridional length of 2.6 in. is chosen on the blade-to-blade surface. It is bounded on the top and bottom by adjacent blade surfaces, and by upstream and downstream boundaries along which the flow is assumed to be uniform. A stream channel is defined by specifying a meridional streamline radius and channel thickness.

Analysis of the gas-particle flowfield was conducted by first determining the magnitude and direction of the gas velocity at all interior grid points of the solution region. The particle velocity vector was then determined by analyzing the path followed by particles in a flowfield with a known gas velocity distribution. The gas flowfield was determined with a solution method<sup>1</sup> which yields a two-dimensional, isentropic, shock-free, transonic flow solution throughout the region. The dynamic behavior of the particles was determined by numerically solving the differential equation of motion of a single spherical particle

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in the absence of potential force fields. The force required to accelerate the particle results from the viscous drag exerted on the particle by the surrounding gas, and is expressed in terms of an empirically determined drag coefficient<sup>2</sup> which corrects the Stokes drag law for rarefaction, compressibility and inertial effects. It was assumed that the particles are noninteracting and uniformly distributed within the gas and do not alter the gas properties.

In order to help establish application guidelines for the LDV technique to the turbomachinery field, the dynamic behavior of several gas-particle mixtures was determined by numerically solving the governing equations for a range of particle sizes and mass densities typically found in LDV gas dynamic applications. The particle radius varied from 0.25  $\mu$  to 2  $\mu$  and the mass density ranged from 1 to 4 g/cc. The gas flow was constant throughout the study with a mass flow rate, inlet mass density, and inlet temperature of  $6.68 \times 10^{-3}$  lb/sec,  $7.403 \times 10^{-2}$  lb/ft<sup>3</sup>, and 523.7 R, respectively. The gas entered the solution region with an average velocity of 285.4 fps and an inlet flow angle of zero relative to the meridional plane. Particles were introduced at several tangential locations along the upstream boundary with a velocity vector essentially the same as the corresponding gas velocity vector. Their trajectories and velocities were numerically determined as they progressed through the region, thereby providing information at all points from which the particle-to-gas velocity ratio and particle angular deviation could be determined.

Results from the gas flowfield analysis indicated that, at any meridional position within the blade channel, the velocity is

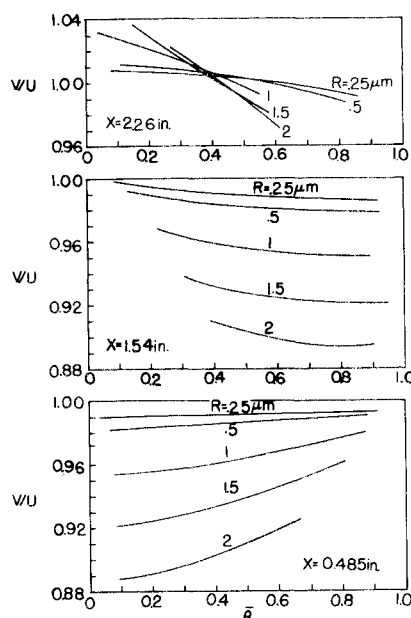


Fig. 1 Tangential variation of particle-to-gas velocity ratio with particle radius as a parameter,  $\rho_p = 1$  g/cc.

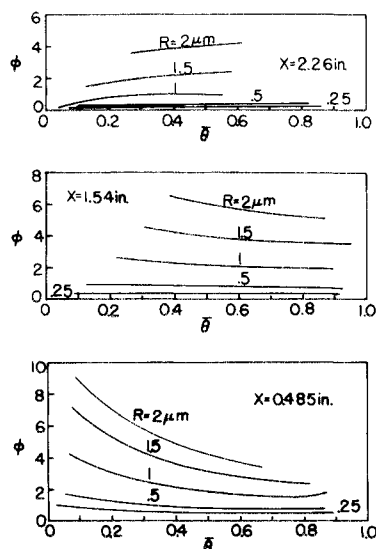


Fig. 2 Tangential variation of angular deviation with radius as a parameter,  $\rho_p = 1 \text{ g/cc}$ .

greatest at the suction surface and decreases to a minimum at the pressure surface, and the flow angle is relatively insensitive to tangential position. The flow is generally well behaved throughout the solution region except for local flow reversals near the trailing edge of the upper blade. The intrablade Mach numbers range from 0.125 to 0.891.

The tangential variation of the particle-to-gas velocity ratio and particle angular deviation at several meridional positions is illustrated in Figs. 1 and 2 as a function of particle radius. The abscissa is the normalized tangential position and is 0 at the suction surface and 1 at the pressure surface. The velocity ratio is defined as the magnitude ratio of the particle velocity to gas velocity  $V/U$ . The particle angular deviation  $\phi$  is defined as  $\phi = |\beta_p - \beta_g|$ . The angles  $\beta_p$  and  $\beta_g$  denote, respectively, the flow angles of the particles and gas measured relative to the meridional direction. LDV applications typically require that  $0.99 \leq V/U \leq 1.01$  and  $\phi < 0.5^\circ$ . The tangential profiles of  $V/U$  and  $\phi$  do not extend the full tangential width of the stream channel because of the channeling of trajectories with increasing meridional streamlength and particle radius. Curves similar to Figs. 1 and 2 plotted as a function of particle mass density are given in the full paper

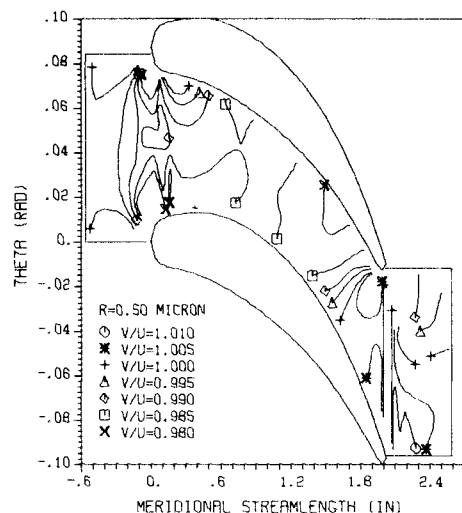


Fig. 3 Contours of constant particle-to-gas velocity ratio,  $\rho_p = 1 \text{ g/cc}$ .

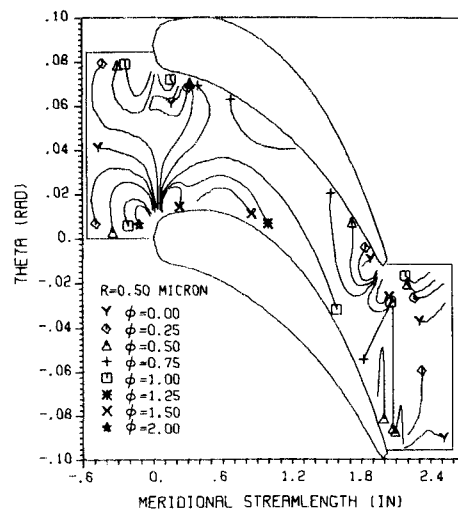


Fig. 4 Contours of constant angular deviation,  $\rho_p = 1 \text{ g/cc}$ .

and indicate that doubling the particle radius causes approximately the same change in  $V/U$  and  $\phi$  throughout the region as a 4 to 1 increase in mass density. Therefore, particle tracking remains essentially unchanged if the particle diameter is halved while at the same time its mass density is increased by a factor of 4. The figures also indicate that tracking is better in the region downstream from the blade channel ( $X = 2.26 \text{ in.}$ ) than in the channel itself. This trend results from the deceleration of the gas in the latter portion of the stream channel and the gradual catching-up of the particles relative to the gas.

Figures 3 and 4 illustrate the tracking capability of  $0.5 \mu$  radius particles through the solution region. Tracking is graphically shown by lines of constant velocity ratio and angular deviation. The regions of greatest velocity lag (high and low  $V/U$ ) occur adjacent to the leading edge of the blades, adjacent to the pressure surface approximately two thirds of the way through the blade channel, and in the region near the trailing edge of the upper blade surface. Minimum velocity lag generally occurs near the entrance to the flow region where the gas-particle mixture was introduced, in the region just prior to the trailing edge of the blades, and in the narrow high gradient region downstream from the channel.

While the results illustrated in Figs. 1–4 are quantitatively applicable to the particle density and radii noted, the results and trends are qualitatively applicable to the entire range of particle properties investigated. The over-all results of the study indicated that in order to achieve velocity and angular deviations generally less than 1% and  $0.5^\circ$ , respectively,  $1 \text{ g/cc}$  tracer particles with diameters of  $0.5 \mu$  or less should be employed. The results also indicated that LDV applications employing  $1 \text{ g/cc}$  tracer particles with diameters greater than approximately  $1 \mu$ , or  $0.5 \mu$ -diam particles with mass densities greater than  $4 \text{ g/cc}$  experience deviations generally greater than 2% and  $1^\circ$ . While these conclusions are based upon high Mach number subsonic flow through a specific blade channel geometry, it is felt they are also approximately valid for high-speed flow through other similar blade configurations with constant meridional streamline radius.

## References

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